Do geodesics exist in the quantum world?

Invited talk by R. Y. Chiao* at the workshop:
"What exists in the quantum world?"
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Our team working on experiments on coherent matter waves interacting with tidal gravity:

- Prof. Kirk Wegter-McNelly, Boston University
- Prof. Michael Scheibner, UC Merced
- Dr. Stephen Minter, Postdoc, UC Merced
- Luis Martinez, graduate student, UC Merced
- Bong-Soo Kang, graduate student, UC Merced

Our goal: To explore the frontier between QM and GR; specifically, to observe the interplay of *coherent* entangled states (e.g., the BCS state) with *tidal* gravitational fields.

Our “Prague” paper

- “Do mirrors for gravitational waves exist?” S.J. Minter, K. Wegter-McNelly, and R.Y. Chiao, Physica E 42 (2010) 234-255.

Concept of "geodesic"

- A *geodesic* is the *classical trajectory* in space-time followed by a freely falling point particle.
- However, Bohr taught us that the very concept of "classical trajectory", e.g., the circular orbit of an electron in the Bohr model of the H atom, is fundamentally *meaningless* in QM.
- Therefore, do geodesics *exist* in the quantum world?

Conceptual tension between *locality* in GR and *non-locality* in QM

- *Localizability* of point particles in GR is assumed in the Equivalence Principle (EP).
- *Non-localizability* of point particles in QM follows from the Uncertainty Principle (UP).
- Are these concepts reconcilable or irreconcilable?
- Are any there experimental consequences of this tension?

Concept of *separability* in GR vs. the concept of *non-separability* in QM

- Einstein in 1935 introduced the term *Trennungsprinzip* (the principle of *separation*) as his most basic physical principle.
- Schrödinger in 1935 introduced the term *Verschränkung* (entanglement) in order to introduce the concept of *non-separability*.
- Violations of Bell’s inequalities show that *non-separability* is an experimental fact of Nature.

Does *non-separability* lead to any consequences for General Relativity?

- Specifically, can we observe any consequences of *entangled states* in *gravitational* physics?
- More specifically, can we observe any new phenomena in *superconducting systems* in the presence of *tidal* gravitational fields?

Why superconductors?

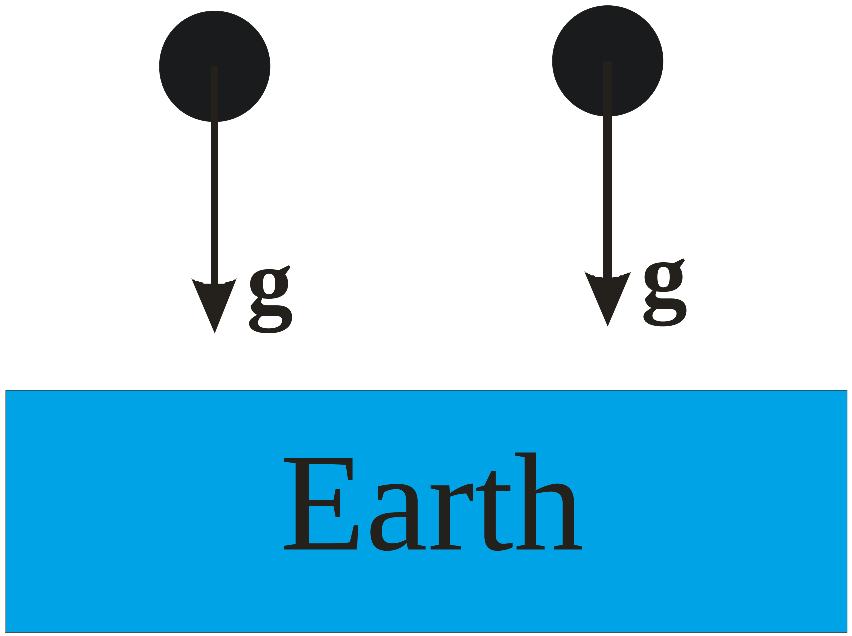
- A Cooper pair of electrons is in the entangled state:

$$\psi_{\mathbf{k}}(\mathbf{r}_1,\mathbf{r}_2,t)=\frac{1}{2}\left(e^{+i\mathbf{k}\cdot\mathbf{r}_1}e^{-i\mathbf{k}\cdot\mathbf{r}_2}+e^{-i\mathbf{k}\cdot\mathbf{r}_1}e^{+i\mathbf{k}\cdot\mathbf{r}_2}\right)\left(\uparrow_1\downarrow_2-\downarrow_1\uparrow_2\right)$$

- Starting from the BCS state, Cooper pairs form a charged analog of the BEC state with a condensate wavefunction (i.e., the “complex order parameter” of Ginzburg and Landau) with a *coherent* phase.

Equivalence Principle:
The *Universality* of Free Fall

- Two dropped objects fall exactly in the same way, *independent* of their *mass*, *composition*, or *quantum state*.
- For example, a spin-singlet *entangled* state and a spin-triplet *entangled* state, i.e., all the Bell states of two objects, undergo free fall in exactly the same way as two *classical* objects fall in a *uniform* gravitational field.
- However, the *universality* of this principle has been precisely tested experimentally only in the case of *uniform* gravitational fields.



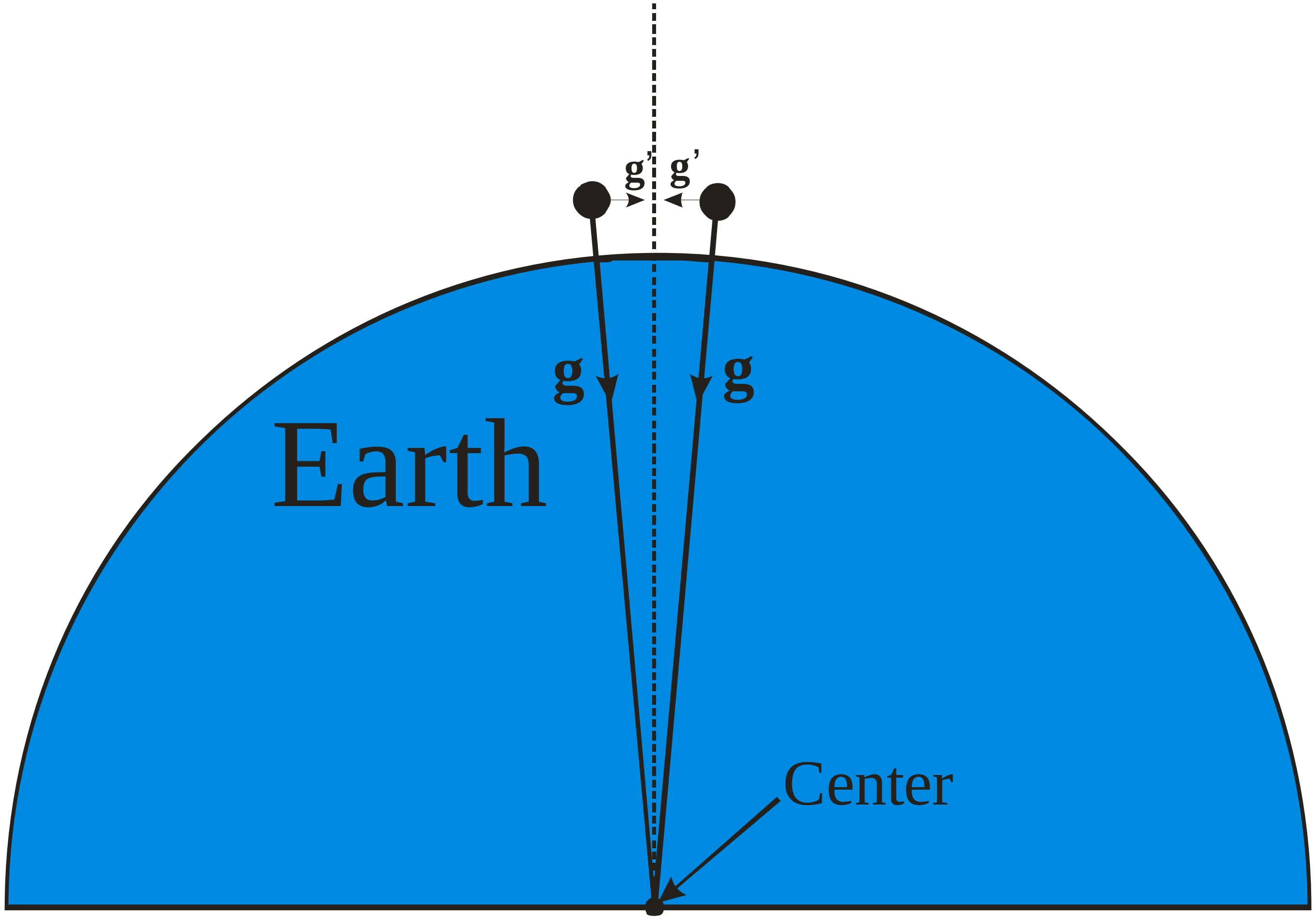
Universality of Free Fall (con’t)

- The principle of the *Universality* of Free Fall seems to transcend all distinctions between *classical* (i.e., incoherent) and *quantum* (i.e., coherent) matter; the superposition principle seems to be transcended by the equivalence principle; coherence or decoherence doesn’t seem to matter in gravitational fields.
- Does this mean that *all* coherent quantum matter waves will always fall in the presence of *all* gravitational fields in *exactly* the same way as *all* classical matter?
- Tests need to be performed in *non-uniform*, i.e., *tidal*, gravitational fields.

Decoherence is required to give meaning to the Equivalence Principle

- ‘The classical concept of *geodesic* depends fundamentally upon the *localizability*, or *spatial separability*, of particles. From a quantum mechanical point of view, *localizability* arises ultimately from the *decoherence* of entangled states, i.e., from the “collapse” of nonfactorizable superpositions of product wavefunctions of two or more particles located at two or more spatially well-separated points in space, into spatially separable, factorizable, product wavefunctions, upon the interaction of the particles with their environment. Decoherence typically occurs on extremely short time-scales due to the slightest interaction with the environment [Zurek]. Whenever it does occur, one can speak classically of point particles having *trajectories* or *traveling along geodesics*. Only after decoherence has occurred does the Equivalence Principle become a well-defined principle, for only then does a particle's geodesic become well defined. In other words, only through decoherence does the law of the “universality of free fall,” i.e., the experimentally well-established claim that “the gravitational acceleration of a point body is independent of its composition” [Adelberger], become meaningful.’ [Quote from our Prague paper with emphasis added]

Simple example of a tidal field: Trajectories of two dropped objects *converge* towards the center of the Earth



Convergence of trajectories means Earth's field is *non-uniform*, i.e., is *tidal*.

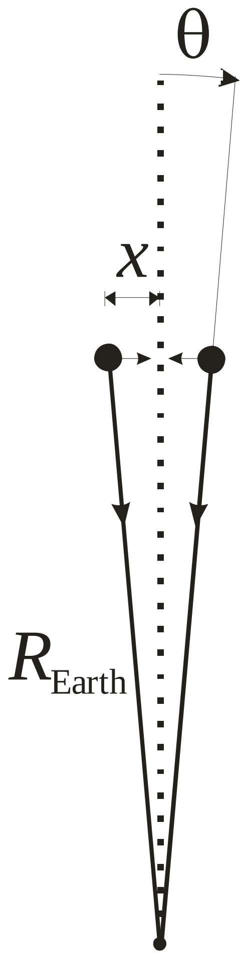
Converging trajectories imply a *tidal* gravitational field for the Earth

- In the freely-falling center-of-mass frame of two dropped, freely falling test particles, horizontally separated by a distance 2*x*, will converge towards each other with a horizontal component of acceleration

$g' \approx g\ \theta,$ where

$\theta \approx \dfrac{x}{R_{\mathrm{Earth}}}$ is

the half-angle of convergence of the trajectories



Some numbers

- Since the radius of the Earth is around 6000 kilometers, for a separation *2x* of the two falling objects around 10 cm, the half-angle of the convergence of their trajectories is around

$$\theta \approx \text{ 10 nanoradians}$$

Effective Hooke's law for horizontal gravitational trapping of particles

- Newton's EOM for the horizontal component of force of a test particle of mass *m* now reads as follows:

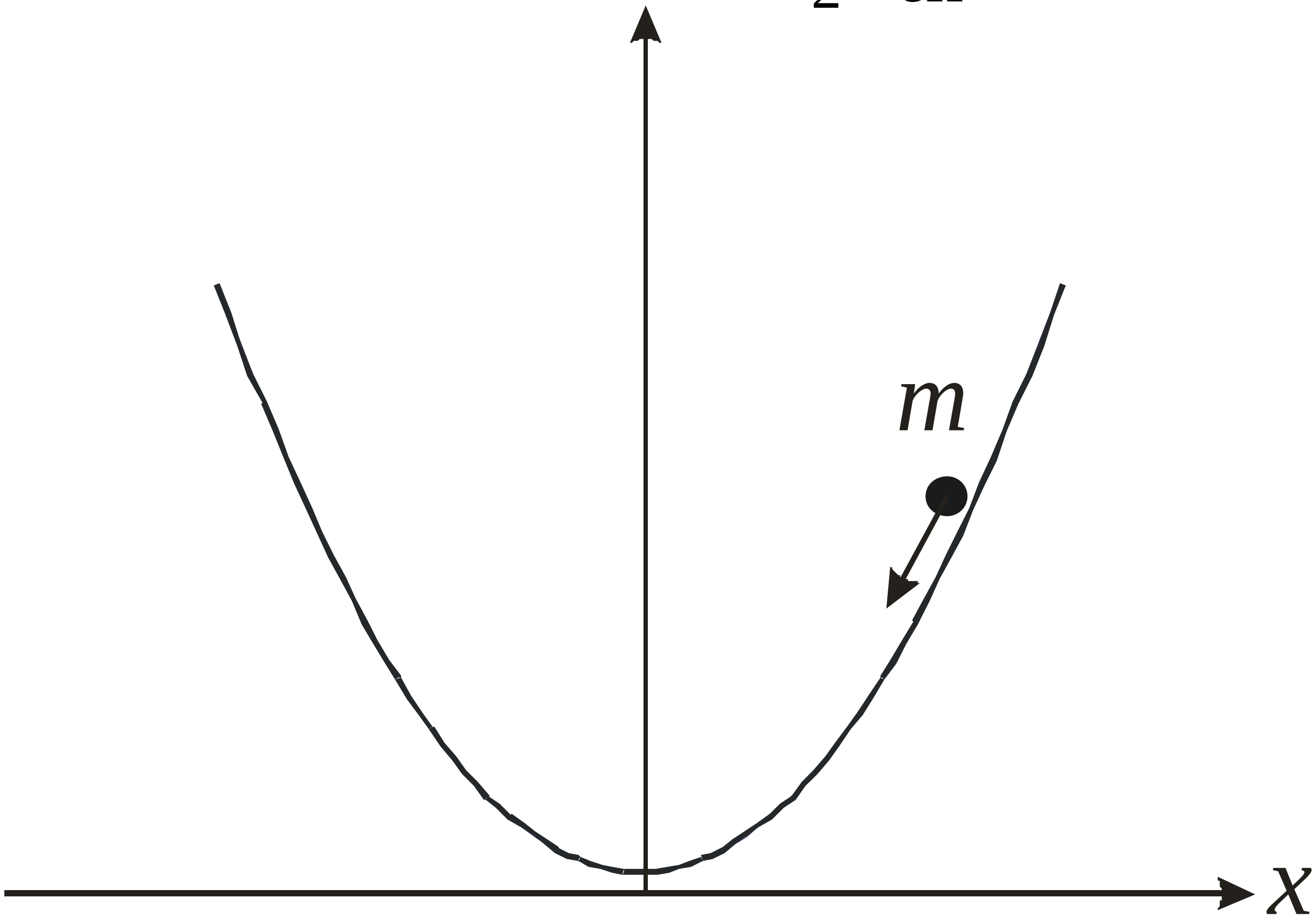
$$F_x = m\frac{d^2x}{dt^2} = -mg\theta = -\left(\frac{mg}{R_{\text{Earth}}}\right)x = -k_{\text{eff}}\,x$$

where $k_{\text{eff}} = \dfrac{mg}{R_{\text{Earth}}}$ is the effective

Hooke's constant.

Effective Hooke's law implies a horizontal tidal trapping potential

$$V(x) = \frac{1}{2} k_{\text{eff}} x^2$$



Connection with general relativity: the Riemann curvature due to Earth’s gravity

The geodesic deviation EOM is

$$\frac{d^2x}{dt^2} = -\left(R_{0x0x}\right)x$$

From the effective Hooke's law,

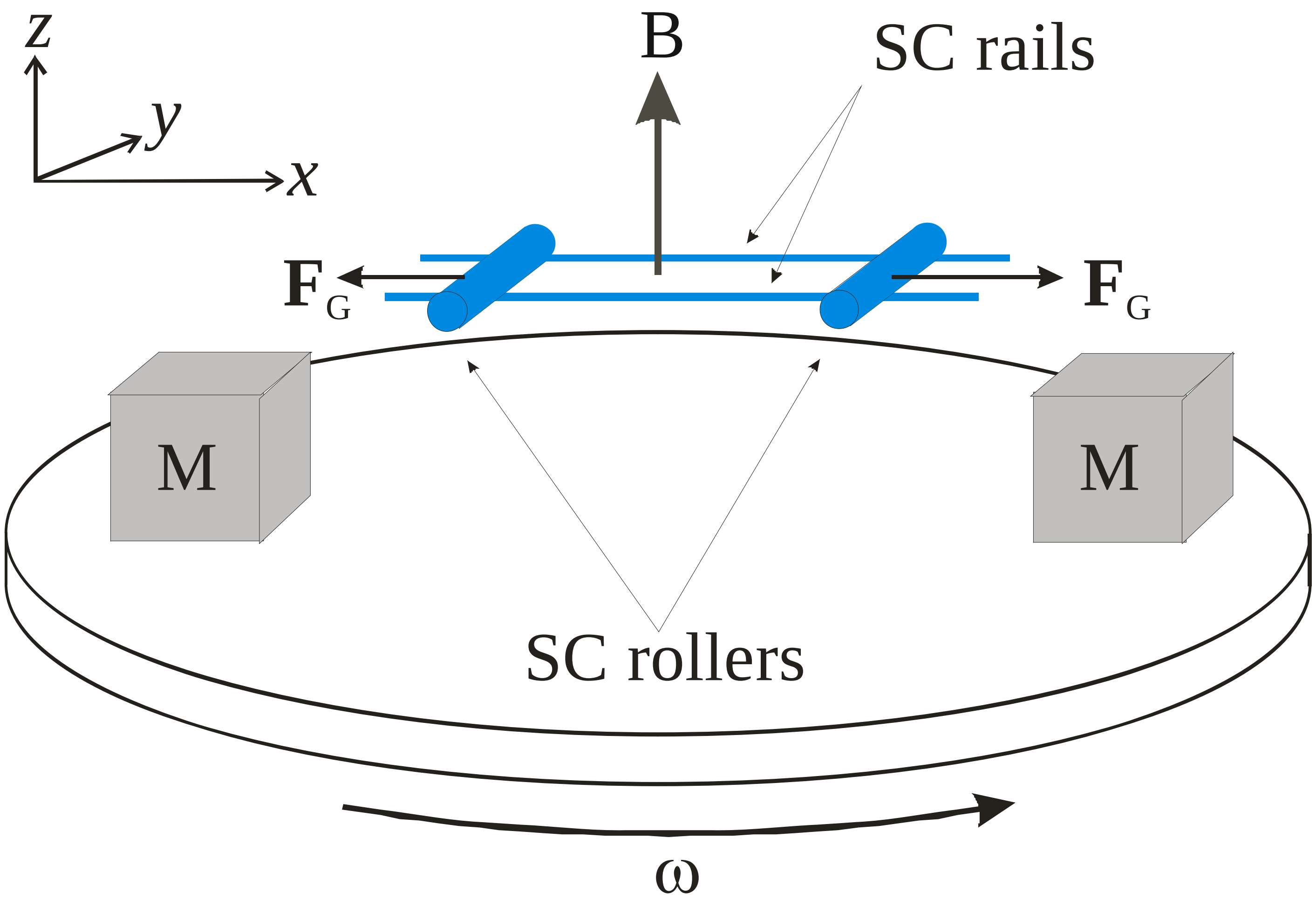
we see that the Riemann tensor

component R_{0x0x} is given by

$$R_{0x0x} = \frac{k_{\text{eff}}}{m} = \frac{g}{R_{\text{Earth}}}$$

which is non-vanishing.

Thought experiment: What is the response of *coherent* quantum matter waves to *tidal* gravity?



Superposition principle applied to the two rollers of thought experiment

- The two rollers are like two moveable slits in Young’s two-slit interference experiment
- It is impossible to know if a given, individual Cooper pair is in the left roller or the right roller.
- If the pair were in the *left* roller, it would feel a *leftwards* gravity force.
- If the pair were in the *right* roller, it would feel a *rightwards* gravity force.
- Superposition: there is a *zero* net force on the pair.
- Therefore, Cooper pairs will remain *motionless* with respect to c.m. of the SC circuit; rollers don’t move.

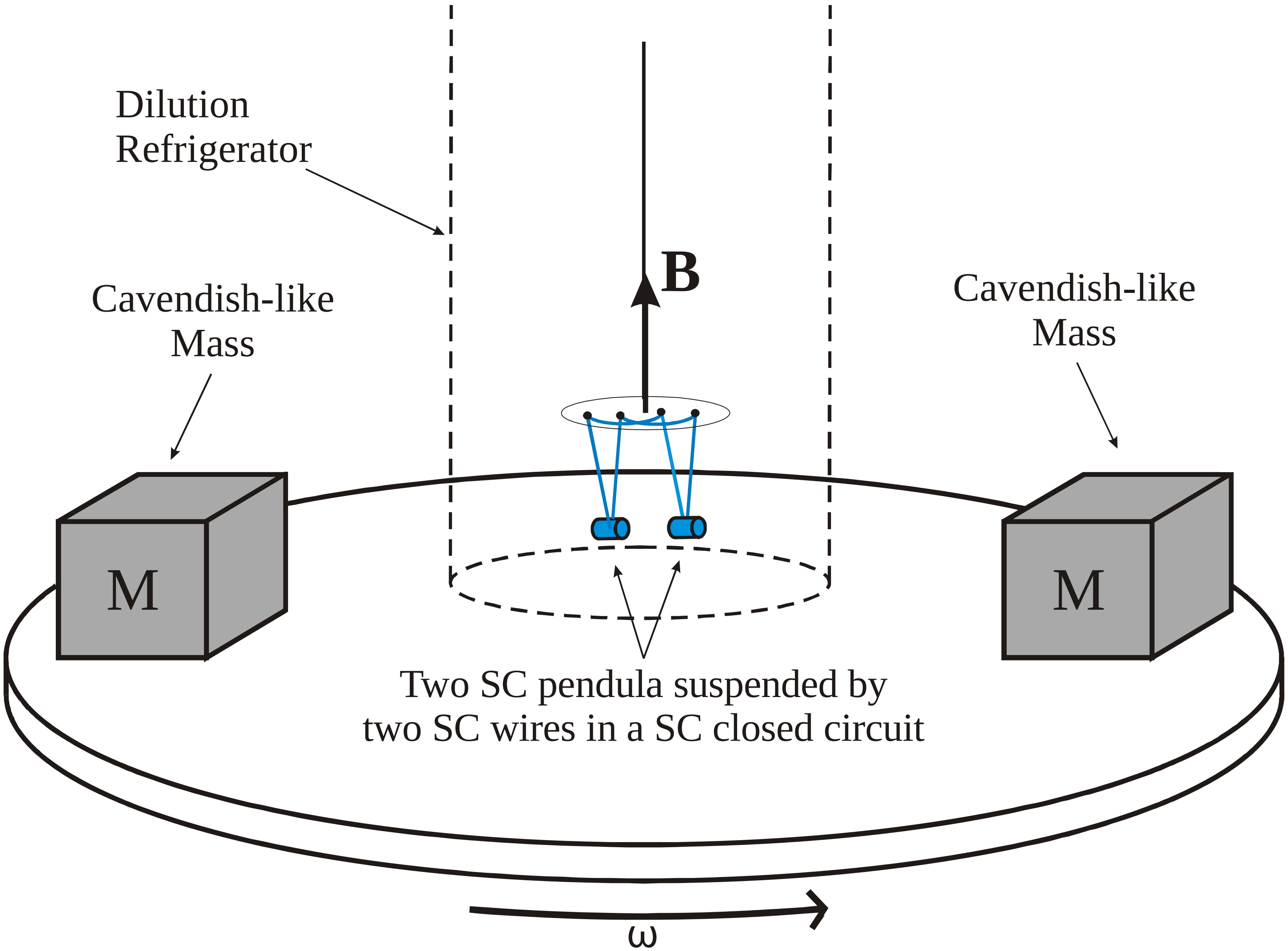
Two possible outcomes of the thought experiment

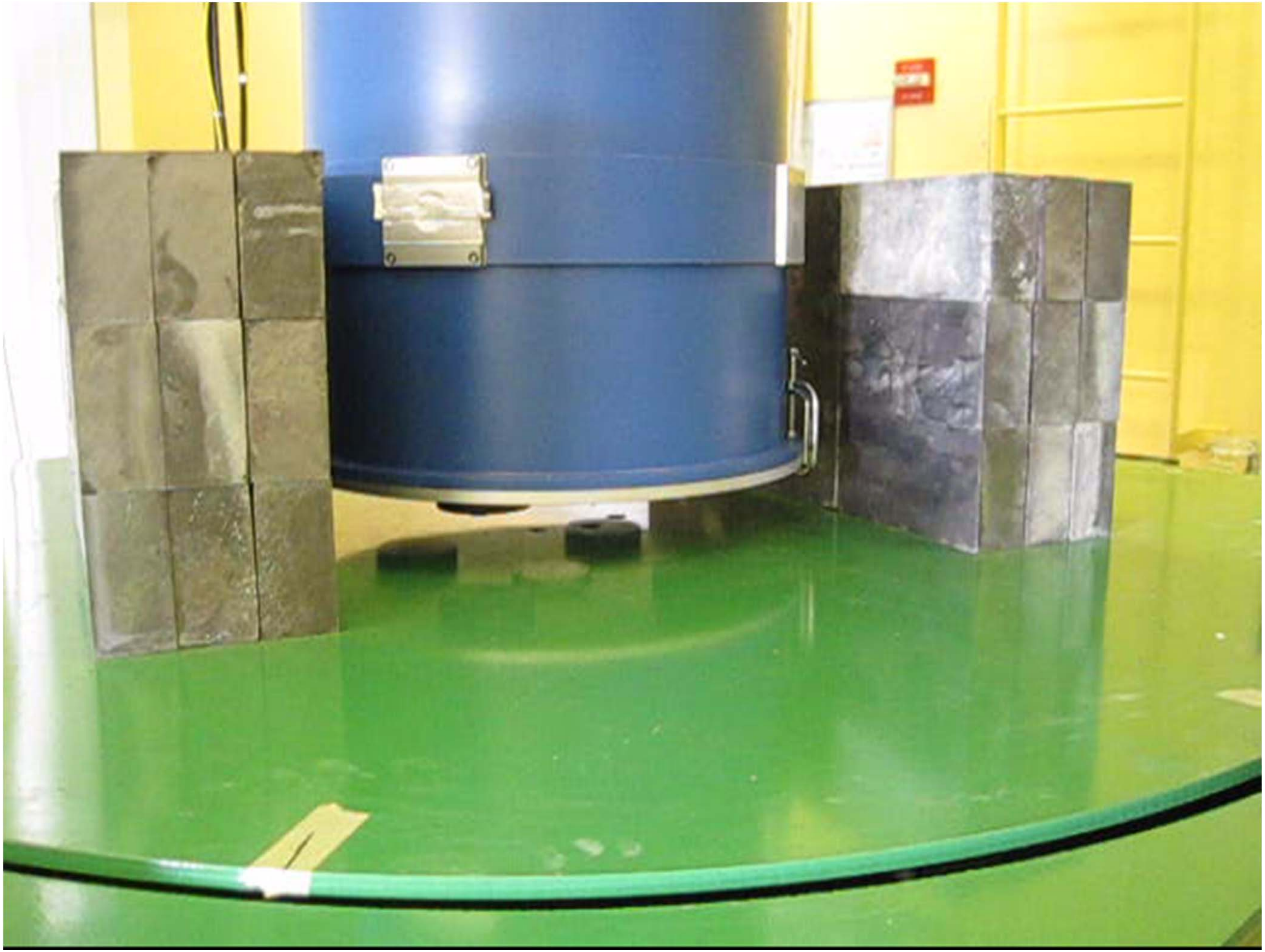
- OUTCOME (I): Below the transition temperature, the rollers stop moving. This implies that the SC system possesses "quantum rigidity."
- OUTCOME (II): Below the transition temperature, the rollers continue to roll back and forth. This implies that the SC system does not possess any "quantum rigidity."

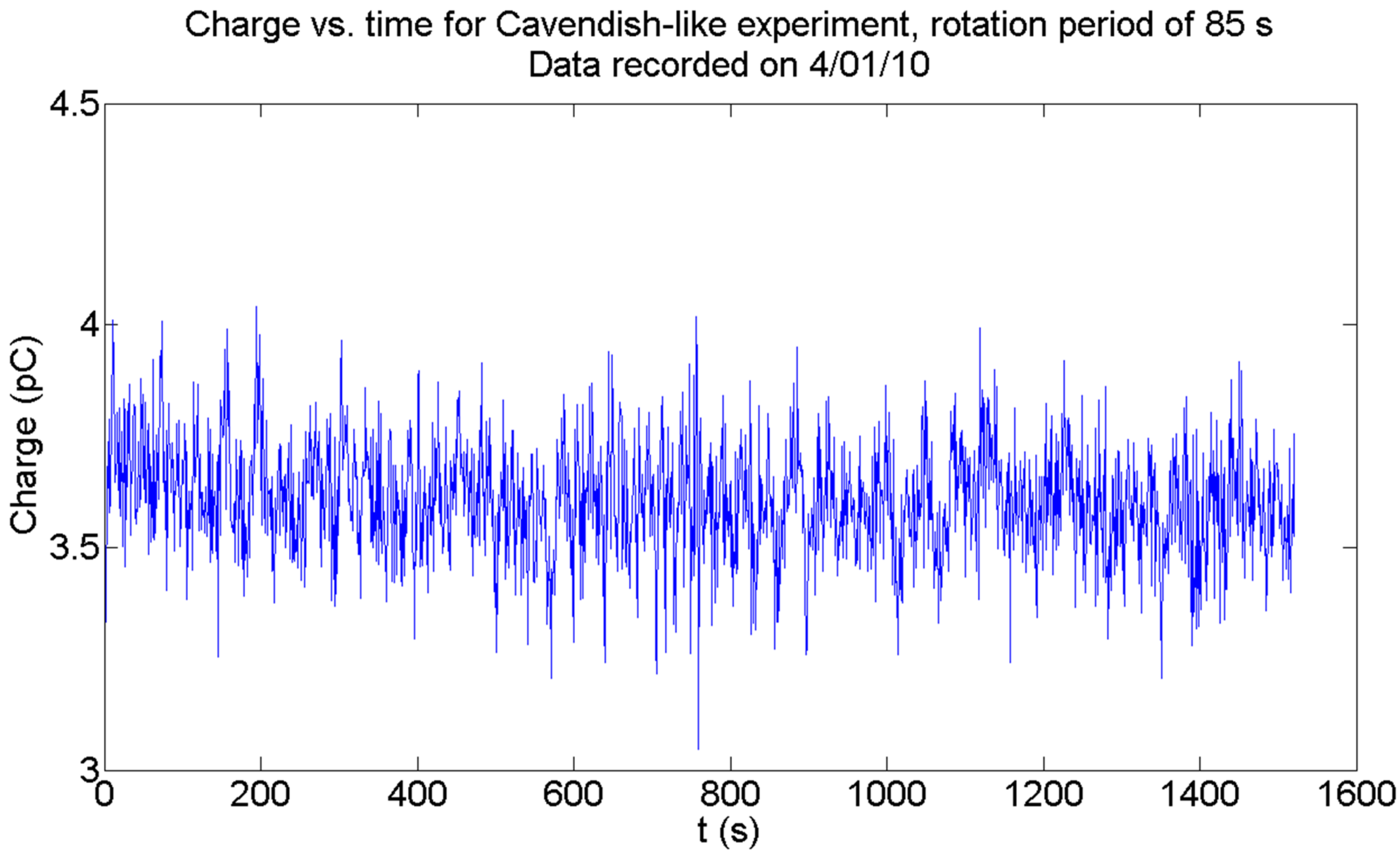
“Charge-separation” effect is implied by OUTCOME (I)

- The ions in the rollers are classical particles.
- They will be attracted to the nearby Cavendish-like source masses.
- The ions will start to move towards these masses.
- The Cooper pairs will remain at rest with respect to the center-of-mass of the circuit.
- Therefore, the ions and Cooper pairs will start to separate from each other.

Cavendish-like experiment using SC pendula





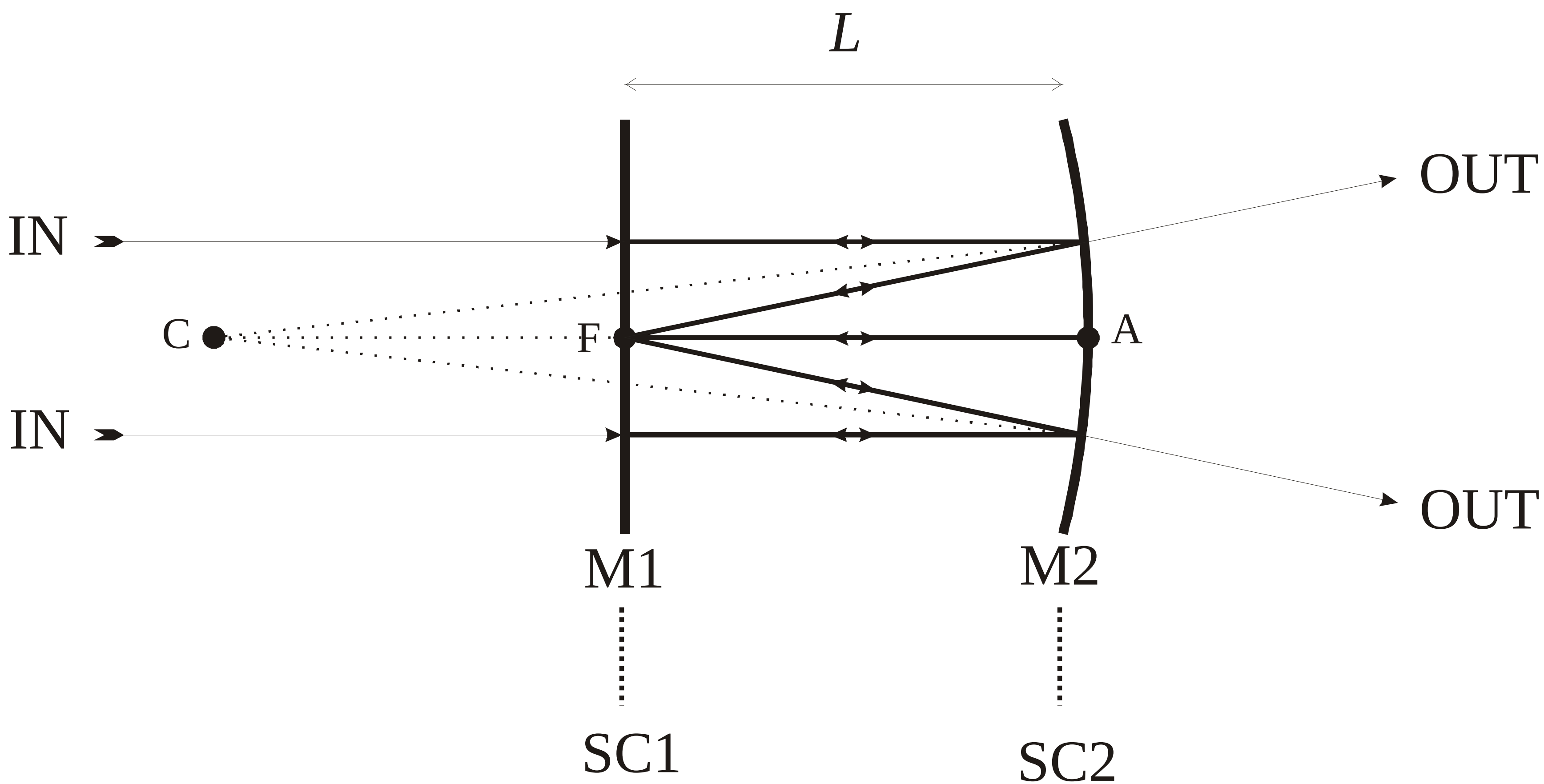

Charge vs. time for Cavendish-like experiment, rotation period of 85 s
Data recorded on 4/01/10
Charge (pC)
t (s)
4.5
4
3.5
3
0
200
400
600
800
1000
1200
1400
1600

The “charge-separation” effect will be nullified by a pair-breaking mechanism

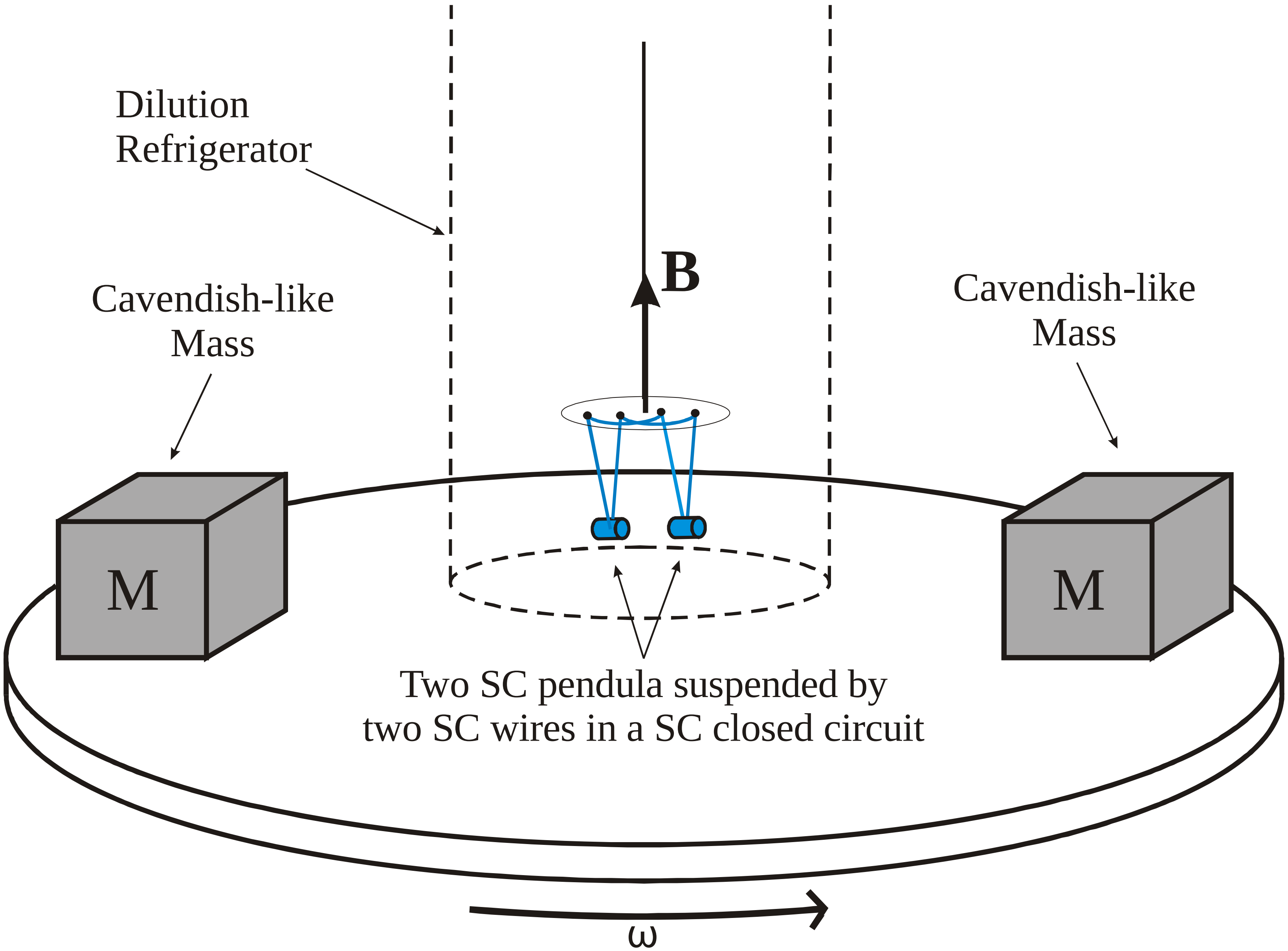
- Luis Martinez pointed out that the internal electric field produced by the charge-separation effect induced by the source masses will accelerate the Cooper pairs to an energy of ~ eVs.
- The binding energy of a Cooper pair is only ~ meVs.
- Therefore, Cooper pairs will break up into normal electrons upon colliding with the surface of the SC, thus nullifying the “charge-separation” effect.

Proposed *optical* Cavendish-like experiment

- A hemiconfocal Fabry-Perot cavity can detect small movements of M2 towards a nearby Cavendish-like mass.



Cavendish-like experiment using SC pendula



Two possible outcomes of the optical Cavendish-like experiment

- OUTCOME (I): Below the transition temperature, mirror M2 stops moving. This implies that the SC system possesses "quantum rigidity."
- OUTCOME (II): Below the transition temperature, mirror M2 continues to move. This implies that the SC system does not possess any "quantum rigidity."

Conclusions

- If OUTCOME (I) is the case, the superposition principle of QM "wins" over the equivalence principle of GR.
- If OUTCOME (II) is the case, the equivalence principle of GR "wins" over the superposition principle of QM .
- More *experiments* are needed at the frontier between QM and GR to settle *existential* matters.