Do geodesics exist in the quantum world?

Invited talk by R. Y. Chiao* at the workshop:
“What exists in the quantum world?”
International Academy Traunkirchen, Austria
19.07.-24.07.2010
*University of California at Merced, U. S. A.
Our team working on experiments on coherent matter waves interacting with tidal gravity:

- Prof. Kirk Wegter-McNelly, Boston University
- Prof. Michael Scheibner, UC Merced
- Dr. Stephen Minter, Postdoc, UC Merced
- Luis Martinez, graduate student, UC Merced
- Bong-Soo Kang, graduate student, UC Merced

Our goal: To explore the frontier between QM and GR; specifically, to observe the interplay of coherent entangled states (e.g., the BCS state) with tidal gravitational fields.
Our “Prague” paper

• “Do mirrors for gravitational waves exist?”
Concept of “geodesic”

- A geodesic is the classical trajectory in space-time followed by a freely falling point particle.
- However, Bohr taught us that the very concept of “classical trajectory”, e.g., the circular orbit of an electron in the Bohr model of the H atom, is fundamentally meaningless in QM.
- Therefore, do geodesics exist in the quantum world?
Conceptual tension between *locality* in GR and *non-locality* in QM

- *Non-localizability* of point particles in QM follows from the Uncertainty Principle (UP).
- Are these concepts reconcilable or irreconcilable?
- Are any there experimental consequences of this tension?
Concept of *separability* in GR vs. the concept of *non-separability* in QM

- Einstein in 1935 introduced the term *Trennungsprinzip* (the principle of *separation*) as his most basic physical principle.
- Schrödinger in 1935 introduced the term *Verschränkung* (entanglement) in order to introduce the concept of *non-separability*.
- Violations of Bell’s inequalities show that *non-separability* is an experimental fact of Nature.
Does non-separability lead to any consequences for General Relativity?

- Specifically, can we observe any consequences of entangled states in gravitational physics?
- More specifically, can we observe any new phenomena in superconducting systems in the presence of tidal gravitational fields?
Why superconductors?

• A Cooper pair of electrons is in the entangled state:

\[ \psi_k(r_1, r_2, t) = \frac{1}{2} \left( e^{ik \cdot r_1} e^{-ik \cdot r_2} + e^{-ik \cdot r_1} e^{ik \cdot r_2} \right) (\uparrow_1 \downarrow_2 - \downarrow_1 \uparrow_2) \]

• Starting from the BCS state, Cooper pairs form a charged analog of the BEC state with a condensate wavefunction (i.e., the “complex order parameter” of Ginzburg and Landau) with a coherent phase.
Equivalence Principle: The *Universality* of Free Fall

- Two dropped objects fall exactly in the same way, *independent* of their *mass*, *composition*, or *quantum state*.
- For example, a spin-singlet *entangled* state and a spin-triplet *entangled* state, i.e., all the Bell states of two objects, undergo free fall in exactly the same way as two *classical* objects fall in a *uniform* gravitational field.
- However, the *universality* of this principle has been precisely tested experimentally only in the case of *uniform* gravitational fields.
Universality of Free Fall (con’t)

• The principle of the *Universality* of Free Fall seems to transcend all distinctions between *classical* (i.e., incoherent) and *quantum* (i.e., coherent) matter; the superposition principle seems to be transcended by the equivalence principle; coherence or decoherence doesn’t seem to matter in gravitational fields.

• Does this mean that *all* coherent quantum matter waves will always fall in the presence of *all* gravitational fields in *exactly* the same way as *all* classical matter?

• Tests need to be performed in *non-uniform*, i.e., *tidal*, gravitational fields.
Decoherence is required to give meaning to the Equivalence Principle

- ‘The classical concept of geodesic depends fundamentally upon the localizability, or spatial separability, of particles. From a quantum mechanical point of view, localizability arises ultimately from the decoherence of entangled states, i.e., from the “collapse” of nonfactorizable superpositions of product wavefunctions of two or more particles located at two or more spatially well-separated points in space, into spatially separable, factorizable, product wavefunctions, upon the interaction of the particles with their environment. Decoherence typically occurs on extremely short time-scales due to the slightest interaction with the environment [Zurek]. Whenever it does occur, one can speak classically of point particles having trajectories or traveling along geodesics. Only after decoherence has occurred does the Equivalence Principle become a well-defined principle, for only then does a particle's geodesic become well defined. In other words, only through decoherence does the law of the “universality of free fall,” i.e., the experimentally well-established claim that “the gravitational acceleration of a point body is independent of its composition” [Adelberger], become meaningful.’ [Quote from our Prague paper with emphasis added]
Simple example of a tidal field: Trajectories of two dropped objects *converge* towards the center of the Earth.

Convergence of trajectories means Earth’s field is *non-uniform*, i.e., is *tidal*. 
Converging trajectories imply a *tidal* gravitational field for the Earth

- In the freely-falling center-of-mass frame of two dropped, freely falling test particles, horizontally separated by a distance $2x$, will converge towards each other with a horizontal component of acceleration $g' \approx g \theta$, where

\[
\theta \approx \frac{x}{R_{\text{Earth}}}
\]

is the half-angle of convergence of the trajectories.
Some numbers

• Since the radius of the Earth is around 6000 kilometers, for a separation $2x$ of the two falling objects around 10 cm, the half-angle of the convergence of their trajectories is around

$$\theta \approx 10 \text{ nanoradians}$$
Effective Hooke’s law for horizontal gravitational trapping of particles

- Newton’s EOM for the horizontal component of force of a test particle of mass $m$ now reads as follows:

$$F_x = m \frac{d^2 x}{dt^2} = -mg \theta = -\left( \frac{mg}{R_{\text{Earth}}} \right) x = -k_{\text{eff}} x$$

where $k_{\text{eff}} = \frac{mg}{R_{\text{Earth}}}$ is the effective Hooke's constant.
Effective Hooke’s law implies a horizontal tidal trapping potential

\[ V(x) = \frac{1}{2} k_{\text{eff}} x^2 \]
Connection with general relativity: the Riemann curvature due to Earth’s gravity

The geodesic deviation EOM is

\[
\frac{d^2 x}{dt^2} = - \left( R_{0x0x} \right) x
\]

From the effective Hooke's law, we see that the Riemann tensor component \( R_{0x0x} \) is given by

\[
R_{0x0x} = \frac{k_{\text{eff}}}{m} = \frac{g}{R_{\text{Earth}}}
\]

which is non-vanishing.
Thought experiment: What is the response of coherent quantum matter waves to tidal gravity?
Superposition principle applied to the two rollers of thought experiment

• The two rollers are like two moveable slits in Young’s two-slit interference experiment
• It is impossible to know if a given, individual Cooper pair is in the left roller or the right roller.
• If the pair were in the left roller, it would feel a leftwards gravity force.
• If the pair were in the right roller, it would feel a rightwards gravity force.
• Superposition: there is a zero net force on the pair.
• Therefore, Cooper pairs will remain motionless with respect to c.m. of the SC circuit; rollers don’t move.
Two possible outcomes of the thought experiment

• OUTCOME (I): Below the transition temperature, the rollers stop moving. This implies that the SC system possesses “quantum rigidity.”

• OUTCOME (II): Below the transition temperature, the rollers continue to roll back and forth. This implies that the SC system does not possess any “quantum rigidity.”
“Charge-separation” effect is implied by OUTCOME (I)

- The ions in the rollers are classical particles.
- They will be attracted to the nearby Cavendish-like source masses.
- The ions will start to move towards these masses.
- The Cooper pairs will remain at rest with respect to the center-of-mass of the circuit.
- Therefore, the ions and Cooper pairs will start to separate from each other.
Cavendish-like experiment using SC pendula

Dilution Refrigerator

Cavendish-like Mass

B

Two SC pendula suspended by two SC wires in a SC closed circuit

Two SC pendula suspended by two SC wires in a SC closed circuit

M

M

ω
Charge vs. time for Cavendish-like experiment, rotation period of 85 s
Data recorded on 4/01/10
The “charge-separation” effect will be nullified by a pair-breaking mechanism

- Luis Martinez pointed out that the internal electric field produced by the charge-separation effect induced by the source masses will accelerate the Cooper pairs to an energy of ~ eVs.
- The binding energy of a Cooper pair is only ~ meVs.
- Therefore, Cooper pairs will break up into normal electrons upon colliding with the surface of the SC, thus nullifying the “charge-separation” effect.
Proposed *optical* Cavendish-like experiment

- A hemiconfocal Fabry-Perot cavity can detect small movements of M2 towards a nearby Cavendish-like mass.
Cavendish-like experiment using SC pendula

- Two SC pendula suspended by two SC wires in a SC closed circuit
- Dilution Refrigerator
- Cavendish-like Mass
- Cavendish-like Mass

\[ \omega \]
Two possible outcomes of the optical Cavendish-like experiment

• OUTCOME (I): Below the transition temperature, mirror M2 stops moving. This implies that the SC system possesses “quantum rigidity.”

• OUTCOME (II): Below the transition temperature, mirror M2 continues to move. This implies that the SC system does not possess any “quantum rigidity.”
Conclusions

• If OUTCOME (I) is the case, the superposition principle of QM “wins” over the equivalence principle of GR.
• If OUTCOME (II) is the case, the equivalence principle of GR “wins” over the superposition principle of QM.
• More experiments are needed at the frontier between QM and GR to settle existential matters.