Features and Events

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Nobody disputes the reality of events, in particular of detection events. Moreover, only those detection events allow us to tell something about properties of quantum systems. The problem is: Are we justified in doing so? Properly speaking, an event tells nothing about anything. An event simply occurs. It has no relation whatever with anything in the past, nor with anything eventually occurring in the future. The delayed–choice experiment has clarified this point in a definitive manner. So, what is what allows us to infer that a certain quantum system has a certain property? We know that many have thought that it is a macroscopic apparatus or even the mind of the observer. These answers are not satisfactory at all, since they are not congruent with a quantum–mechanical treatment of reality. Moreover, they are not sufficiently accurate. We know very well that what happens before the measurement is a *coupling* between the object system on which we performa measurement and an apparatus. This is also called a premeasurement. Now, what a coupling consists of? In establishing a correlation between object system and apparatus. This correlation in quantum mechanics is called an entanglement. Let us describe in a general way what happens during the whole process starting with the initial preparation of the system up to the detection event. We assume that the initial state of the apparatus is some ready–state $|\, A_0\rangle$ while the state of the object system is the superposition

$$|\, \psi\rangle = \sum_k c_k \,|\, k\rangle \quad . \tag{1}$$

We may describe the process in terms of the (initially factorized) density matrices:

$$\hat{\rho}^A = |\, A_0\rangle \langle A_0 \,| \quad \text{and} \quad \hat{\rho}^S = |\, \psi\rangle \langle \psi \,| \ . \tag{2}$$

Then, we have:

$$\hat{\rho}^S \hat{\rho}^A \mapsto \hat{U}_t \left(\hat{\rho}^S \hat{\rho}^A\right) \hat{U}_t^\dagger \ . \tag{3}$$

To be easy, in the case of a bidimensional system we have:

$$\begin{aligned} \hat{U}_t \left(\hat{\rho}^S \hat{\rho}^A\right) \hat{U}_t^\dagger \quad = \quad & |c_m|^2 \,|\, m, a_m\rangle \langle m, a_m \,| + |c_n|^2 \,|\, n, a_n\rangle \langle n, a_n \,| \\ & + c_m c_n^* \,|\, m, a_m\rangle \ \langle n, a_n \,| + c_m^* c_n \,|\, n, a_n\rangle \ \langle m, a_m \,| \ . \end{aligned} \tag{4}$$

Just before the detection, the probability distribution to read the value a_m of the apparatus observable will be simply given by

$$\wp(a_m) = \mathrm{Tr}_{\mathcal{A}} \left[|a_m\rangle\langle a_m| \mathrm{Tr}_{\mathcal{S}} \left(\hat{U}_t \hat{\rho}^{\mathcal{S}} \hat{\rho}^{\mathcal{A}} \hat{U}_t^\dagger \right) \right] \ . \tag{5}$$

Indeed, computation of the partial trace:

$$\mathrm{Tr}_{\mathcal{S}} \left(\hat{U}_t \hat{\rho}^{\mathcal{S}} \hat{\rho}^{\mathcal{A}} \hat{U}_t^\dagger \right) \tag{6}$$

will kill all of the system's states in (4). By applying the projector $\hat{P}_{a_m} = |\, a_m\rangle \langle a_m \,|$ to the previous result, we shall obtain

$$\hat{P}_{a_m} \mathrm{Tr}_{\mathcal{S}} \left(\hat{U}_t \hat{\rho}^{\mathcal{S}} \hat{\rho}^{\mathcal{A}} \hat{U}_t^\dagger \right) = |c_m|^2 \,|\, a_m\rangle \langle a_m \,| \ , \tag{7}$$

and by finally tracing out the apparatus, we shall get the probability, which, in our case, is $|c_m|^2$. Let us now apply the cyclic property of the trace, so that we obtain

$$\mathrm{Tr}_{\mathcal{A}} \left[|a_m\rangle\langle a_m| \mathrm{Tr}_{\mathcal{S}} \left(\hat{U}_t \hat{\rho}^{\mathcal{S}} \hat{\rho}^{\mathcal{A}} \hat{U}_t^\dagger \right) \right] = \mathrm{Tr}_{\mathcal{A}} \left[\mathrm{Tr}_{\mathcal{S}} \left(\hat{U}_t^\dagger |a_m\rangle\langle a_m| \hat{U}_t \hat{\rho}^{\mathcal{S}} \hat{\rho}^{\mathcal{A}} \right) \right] \ . \tag{8}$$

The reason for extracting $\mathrm{Tr}_{\mathcal{S}}$ is that it does not act on the projector. For the reason that this partial trace does not act on $\hat{\rho}^{A}$, we can rewrite last equation as

$$\mathrm{Tr}_{\mathcal{A}}\left[\mathrm{Tr}_{\mathcal{S}}\left(\hat{U}_t^{\dagger}|a_m\rangle\langle a_m|\hat{U}_t\hat{\rho}^{\mathcal{S}}\hat{\rho}^{\mathcal{A}}\right)\right] = \mathrm{Tr}_{\mathcal{A}}\left[\mathrm{Tr}_{\mathcal{S}}\left(\hat{U}_t^{\dagger}|a_m\rangle\langle a_m|\hat{U}_t\hat{\rho}^{\mathcal{S}}\right)\hat{\rho}^{\mathcal{A}}\right] . \tag{9}$$

The expression

$$\mathrm{Tr}_{\mathcal{S}}\left(\hat{U}_t^{\dagger}|a_m\rangle\langle a_m|\hat{U}_t\hat{\rho}^{\mathcal{S}}\right) = \hat{E}_{a_m} \tag{10}$$

is a projection–like operator, called effect, that does not satisfy the requirement of orthogonality. This allows us to write the above probability as

$$\wp(a_m) = \mathrm{Tr}_{\mathcal{A}}\left[\hat{E}(a_m)\hat{\rho}_i^{\mathcal{A}}\right] , \tag{11}$$

When, we compute the effect explicitly, it gives

$$\begin{aligned}
\hat{E}_{a_m} &= \mathrm{Tr}_{\mathcal{S}}\left(\hat{U}_t^{\dagger}|a_m\rangle\langle a_m|\hat{U}_t\hat{\rho}^{\mathcal{S}}\right) \\
&= \left\langle\psi\left|\left(\hat{U}_t^{\dagger}|a_m\rangle\langle a_m|\hat{U}_t\,|\psi\rangle\,\langle\psi|\right)\right|\psi\right\rangle \\
&= \left\langle\psi\left|\hat{U}_t^{\dagger}|a_m\rangle\langle a_m|\hat{U}_t\right|\psi\right\rangle \\
&= \hat{\vartheta}^{\dagger}(a_m)\hat{\vartheta}(a_m) ,
\end{aligned} \tag{12}$$

where

$$\hat{\vartheta}(a_m) = \langle a_m|\hat{U}_t|\psi\rangle . \tag{13}$$

These are not probability amplitudes because the involved unitary operators represents the coupling of the apparatus and the system, whereas the ket and the bra belong to the apparatus' or system's Hilbert space only. As a result, it represents an amplitude *operator*. As we have said, this amplitude operator describes all steps of the measurement of a given observable: Preparation of the initial state of the system ($|\psi\rangle$), unitary evolution (coupling or premeasurement) of the apparatus together with the object system ($\hat{U}_t$), and detection by the apparatus ($\langle a_m|$).

Summarizing, it is thanks to correlations allowing us to pair object system's state components and apparatus' state components that we are finally allowed to ascribe properties to the former. It is now difficult to believe that we can do this without being correlations some form of reality. Obviously, they cannot be taken to be real as detection events are, since events are well localized happenings by definition. Instead, correlations are known to be non–local (to violate the separability principle of classical mechanics). Moreover, correlations cannot be measured or experienced directly. Their existence can only be inferred when we compare the statistics of several quantum systems prepared in the same state. In this sense, although they determine this final statistics and therefore must be taken as a character of the state, they are not properties in the genuine sense of the word, since properties are local by definition. Since such non–local quantum phenomena appear in diverse situations and contexts and even with diverse names, for instance in a superposition of an isolated quantum system and not only in multi–particle entanglement, let us call them, with a single word, *features*.

Now the issue is which kind of reality features represent. My guess is that they are a potential form of reality. Features are potential realities as far as:

- They represent a necessary ingredient of any quantum–mechanical interaction process (they play a key role in premeasurement, as we shall see).
- When activated, they contribute to the actualization of possible outcomes.
- Some actual environmental conditions (including apparata) are also necessary for one of these outcomes to occur, resulting in some event.

In other words, features are *activated* in certain contexts to give rise to certain consequences, thus showing the capability of contributing to determine actual reality. Summarizing, to use the term *feature* has a double advantage: It stresses that we deal with a reality (although potential) and it may be used to address all of the different quantum non–local phenomena.

INPUT		OUPUT
p	q	$\neg p \vee \neg q$
1	1	0
1	0	1
0	1	1
0	0	1

Table 1: Disjunction truth table between $\neg p$ and $\neg q$.

One may obviously worry about the concept of potentiality as such. However, this worry is again not particularly supported by facts. It is very common in physics to speak (in the presence of some field) of potential energy as a potentiality to do work in appropriate conditions (for instance, a stone that can roll down a hill due to the effect of the gravitational field). It is true that one could object that this is only due to the effect of the external field and therefore has nothing to do with the character of the object as such. This objection does not seem very sound to me however; moreover, we have wonderful examples of intrinsic potentiality, if this word can pass here. For instance, in chemistry the chemical potential is proportional to the Gibbs free energy and expresses the potential of a chemical substance for undergoing a change in a system. This is interesting because the Gibbs free energy expresses potentiality to do work in pure chemical terms (that is, without considering the work performed when a system expands in the presence of an opposite external force). I recall that, at equilibrium, the chemical potential of a substance is the same throughout a sample, regardless of how many phases are present.

Now, if we admit that they are a potential form of reality, what consist this potentiality of? My guess is that they are a form of information. Indeed, they represent an additional source of information that can be used (made active) in certain contexts like teleportation. In general, the state of a quantum system represents information codification. Indeed, the basis in which we expand a quantum state is a set of elementary units that can be combined according to syntactic rules (the coefficients) and we have several codices (the different bases) as well as (unitary) transformation rules from one codex to another.

Now, let us go back to the issue of measurement. We can account very well measurement as a decoherence process. Many have seen in such an explanation a practical trick (a FAPP expedient) especially taking into consideration its local and smooth character. We think on the contrary that these are its strengths. The smoothness (the fact that features are never totally washed out) is well confirmed today by Schrödinger–cat–like experiments at a mesoscopic level. About locality, a theorem of quantum mechanics tells us that what is locally irreversible process can be embedded in a larger context that is reversible.

Indeed, it is always possible to make use of some logical gate that is expanded to other propositions and that is logically equivalent to an irreversible logical operation being reversible itself. Since quantum systems consists of information they can also be considered as information processors across the time. For instance, let us consider the inclusive disjunction between the negation of p and and the negation of q as shown in the truth table 1. The reason, for this truth table is that p is true when its negation $\neg p$ is false and vice versa. Nevertheless, as we can see, the form of this truth value is still irreversible. I also note that $\neg p \vee \neg q$ is logically equivalent to $\neg(p \wedge q)$ (by the so called De Morgan's law). The latter expression is called NOT AND.

Let us consider now not just two propositions but enlarge the set of propositions to three, p, q, r. For our purposes we make use of the so–called Toffoli gate shown in Tab. 2. I recall that the exclusive disjunction (XOR) on the last column on the right

- Is true only when
  - **(1)** Either $r = 1$ and $(p \wedge q) = 0$
  - **(2)** Or $r = 0$ and $(p \wedge q) = 1$.
- In the other two cases, it is false i.e. when
  - **(3)** Either both r and $p \wedge q$ are true
  - **(4)** Or both r and $p \wedge q$ are false.

INPUT			OUTPUT		
p	q	r	p	q	$r \succ\!\!\prec (p \wedge q)$
0	0	0	0	0	0
0	0	1	0	0	1
0	1	0	0	1	0
0	1	1	0	1	1
1	0	0	1	0	0
1	0	1	1	0	1
1	1	0	1	1	1
1	1	1	1	1	0

Table 2: Toffoli truth table. p, q are the control bits and are not changed under the transformation.

INPUT		OUPUT
p	q	$p \wedge q$
1	1	1
1	0	0
0	1	0
0	0	0

Table 3: Conjunction truth table.

If we take into account the fact that the conjunction between p and q is false in three cases according to the truth table 3, we have the following inputs represented by the three first columns on the left:

1. Three cases when $r = 1$ and $(p \wedge q) = 0$ (the second, forth and sixth rows).
2. The seventh row corresponds to the case $r = 0$ and $(p \wedge q) = 1$.
3. The latter row corresponds to $r = 1$ and $(p \wedge q) = 1$.
4. We have finally three cases when $r = 0$ and $(p \wedge q) = 0$ (the first, third, and fifth rows).

Taking into account these inputs, let us consider the target output represented by the last column on the right:

1. All the values on the second, fourth and sixth row of the last column are 1, since every time here $r = 1$ and $(p \wedge q) = 0$.
2. The value on the seventh row of the last column is again 1 since $r = 0$ and $(p \wedge q) = 1$.
3. The value shown in the last row on the last column is 0 since both r and $p \wedge q$ are true.
4. The values shown in first, third and fifth rows on the last column on the right are all 0 since both r and $p \wedge q$ are false.

Therefore, if we only consider the cases in which both $r = 1$ and $[r \succ\!\!\prec (p \wedge q)] = 1$, we have precisely those cases in which $p \wedge q$ is false (second, third and fifth columns). This can be expressed as

$$1 \succ\!\!\prec \neg(p \wedge q) \quad \text{or also} \quad 1 \succ\!\!\prec (\neg p \vee \neg q)\ . \tag{14}$$

Therefore we have obtained the inclusive disjunction (OR) between $\neg p$ and $\neg q$, which in itself is an irreversible gate but as embedded in a larger reversible transformation. Note indeed that all outputs of the Toffoli gate are univocally mapped to the inputs: all truth values of p and q are maintained, while the values of r and $r \succ\!\!\prec (p \wedge q)$ are also the same apart the last two lines (when both p and q are true) whose truth value is inverted.

This transformation can be also described quantum–mechanically. The propositions p, q, r can be represented by three different systems that can be each in the state $|1\rangle$ or $|0\rangle$, corresponding to truth values 1 and 0, respectively, so that we can write

$$\begin{aligned} |p\rangle \otimes |q\rangle \otimes |r\rangle &= |pqr\rangle \\ &= |000\rangle + |001\rangle + |010\rangle + |011\rangle + |100\rangle + |101\rangle + |110\rangle + |111\rangle \ , \end{aligned} \tag{15}$$

where for the sake of simplicity I have not considered the coefficients and have shrunk the three vectors in a single one. Therefore, all the above $2^3 = 8$ combinations shown in the first three columns of Tab. 2 can be represented by the 8–row and 3-column vector

$$|pqr\rangle = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix} \tag{16}$$

where each row univocally corresponds to one of the 8 states above. The Toffoli transformation 2 can be represented by a matrix so that we finally obtain

$$\begin{matrix} \textbf{TOFFOLI GATE} & \textbf{INPUT} & & \textbf{OUTPUT} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} & \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix} & = & \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix} \end{matrix} , \tag{17}$$

where it is understood that the Toffoli matrix first multiplies the first input column on the left giving rise to the first output column on the left, then the second input column giving rise to the second output column and finally the third input column giving rise to the third output column.

This means that we have entropy growth only at a local level, while, when considering interaction processes in a larger context, they are reversible, as predicted by quantum mechanics. In particular, if the larger system is a pure state, it must be in a zero–entropy state. If this is true and if the universe can be described quantum–mechanically, we can assume that the universe is in a zero–entropy state. Then, entropy–growing processes are only local and correspond to a spontaneous tendency of every open physical system to disorder. However, if the universe as a whole obeys to quantum mechanical laws (as we are inclined to think), we would expect that the tendency to disorder is continuously balanced by a (entropy decreasing) tendency to order, so that the net result is precisely zero. The conservation of both order and disorder in our universe is likely a far more general principle than the conservation of physical quantities like mass, energy, momentum. Obviously, the tendency to disorder is spontaneous and in this sense more fundamental. Indeed, according to statistical mechanics according the possible disordered states of a system are much more than the ordered ones (for instance, there are likely infinite ways to break a cup, but only few to build it). The crucial point is that every time such a tendency manifests itself, also a *compensatory* tendency to order will be produced to preserve this net balance [Fig. 1]. This second tendency can be said to be less fundamental and not spontaneous, that is, forced by the first one. A consequence of these considerations, is that the tendency to order (as beautifully displayed by all manifestations of life) will also be displayed through growing levels of complexification, without resorting to any form of vitalism. When a more complex reality emerges from a less complex on, the immediate result is that we have a larger number of components or factors that are integrated in the new system. Clear evidence for that is the higher number of elements and interrelations in a bio–molecule like a protein relative to any abiotic molecule like water. In this

Figure 1: A constant–entropy (perhaps a zero–entropy) universe in which local disruptive and order–building processes do not affect the configuration of the whole.

way, getting the more complex from the less complex implies a growing number of constraints and therefore also a lower probability to find the ordered or stable configuration. In other words, the number of possible disordered (and unstable) configurations grows exponentially with complexification. This implies that all bio–molecules, for instance, share much more constraints among them than all abiotic ones do. This explains that along the process of growing complexification there is also a sort of growing canalization,Canalization where this statement should not be taken in the sense that we have less and less variety, but precisely in the sense that we have more and more shared constraints.

Let us go back again to measurement. A quantum system needs beforehand to be prepared. A preparation can be understood as a *determination* of the *state* of a single *system*. It is the procedure through which only *systems in a certain* (previously theoretically defined) *state are selected* and delivered for further procedures, that is, allowed to undergo subsequent operations (premeasurement and measurement).

A premasurement consists in an *interrogation* of a quantum system relative to a specific degree of freedom. Quantum mechanics seems to imply that the specific basis used for the expansion of the compound state of the apparatus and the system is irrelevant, and therefore that premeasurement is not about a specific observable. The fact is that, at a rather abstract level, several bases for a state of two coupled systems are possible. However, we should not mix measurement *procedures with algorithms*. When we consider a specific physical situation (that is, once a premeasurement is done), we introduce a further degree of determination and are no longer authorized to treat different experimental contexts as equivalent.

Therefore, it would be highly unphysical to consider all observables as equivalent *in a concrete experimental context* since, changing the apparatus' basis means a concrete change in the apparatus as such, so that we may no longer assume to have to do with the same or an *equivalent* measurement process. For this reason, choosing a *certain* experimental context univocally individuates a *certain* observable. This is exactly the reason why we said that *actual* external conditions are needed to obtain an event. That an experimental context individuates a certain observable (better, a certain degree of freedom) is also true, to a certain extent, from a classical point of view, since each apparatus is better suitable for measuring a certain observable and not others.

A measurement (detection) is an *answer* to our interrogation. When we establish an entanglement, we are actually also entangling the object system with some detectors. Although detectors are in general considered as part of an apparatus, they are conceptually very different. An apparatus is a *coupling* device, while a detector is a *selection* device. This justifies the fact that, properly speaking, the apparatus is an interface between detector and object system.

When a suitable selection is made, the detection apparatus is in one of its basis states and, upon the coupling with the object system, it tells information about the latter. This connection allows for a certain random outcome to tell us something about the input state.

On this basis, as already announced, we may consider that each step here (from preparation through premeasurement up to detection) can be considered as a further degree of determination, or the whole can be seen as a dynamical and local process through which, starting from some potential reality and suitable actual conditions, an actual reality (the event) is obtained.

In conclusion, the concept of existence in quantum mechanics must be fine–tuned. We cannot ascribe existence to any phenomenon in the same way. Events exist in an immediate (actual) sense of the word. However, also features exist, although in a mediate and potential sense of the word. Finally, states, observables and properties also are real but in another sense. They are equivalence classes and so appears rather as concepts than as real things. They certainly are not things. However, they are what could be called an interpreted piece of ontology (in the framework of the theory that we call quantum mechanics) that is nevertheless referred to an unintepreted one: what we have called events and features. Indeed, properties are equivalence classes *of detection events*, while observables are equivalence classes *of couplings* and therefore of certain kinds of features. So, those concepts are finally rooted in an ontology.